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## Line Array Performance at Mid and High Frequencies

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### ABSTRACT

The paper is focusing on the direct sound frequency response of line arrays - rectilinear or curved - at mid and high frequencies (1kHz – 10kHz) which is arguably the most important range and one that is relatively easy to measure. In this frequency range a line array may produce irregular on- and off-axis frequency responses in the audience area, which is difficult to predict using simpler models. The irregularities, which appear as frequency varying attenuation, depend in a complicated way on array configuration and air absorption.

Array performance prediction software usually models a line array as a number of directive point sources placed on a line or curve. The directive point source model has been used to simulate line arrays to study the frequency response behaviour of line arrays at mid and high frequencies. The results of the study are compared with frequency response predictions calculated by new software including multi-channel array controller simulations and measured complex spherical polar data for a specific 3-way line array cabinet. The predictions are compared to direct sound frequency response measurements on line arrays using the same 3-way cabinet to show the degree of accuracy with which directive point source models can predict the frequency responses of line arrays.

### 1. INTRODUCTION

Most papers on line array analysis have studied the directivity patterns of line arrays, see for example [1,2,3]. This paper will focus on another important aspect of line arrays, that is, the direct sound mid and high frequency responses they produce in the audience area. The irregularities, that line arrays may show in their direct sound frequency responses appear as a

frequency varying attenuation in the mid and high frequency range. The attenuation depends in a complicated way on the number of cabinets in the array, the length and curvature of the array, the off-axis angle, the distance from the array to the frequency response observation point in the audience area, the way the cabinets in the array are driven and air absorption.

Optimization of the control of line array performance is today offered by software using measured complex spherical polar data for the array modules. The prediction software is basically modeling a line array as a number of directive point sources placed on a line or curve. The directive point source model has been used here to simulate line arrays for a study of their direct sound mid and high frequencies responses in the audience area. Instead of using measured complex spherical polar data each cabinet in the array is modeled as a point source having a frequency response equal to 1, a specified sensitivity, a specified phase response and a specified directivity function (directive point source model 1).

The directive point source model will make it possible to study how mid and high frequency attenuation varies with the number of cabinets in the array, the length and curvature of the array, the directivity of the array cabinets, the distance from the array to the frequency response observation point in the audience area, the off-axis angles corresponding to the observation point and the frequency, humidity and pressure dependent air attenuation. Only direct sound frequency responses have been studied here. The added effects of room-acoustical factors such as reflections and reverberation have not been considered.

Array frequency response predictions calculated by new software (directive point source model 2) including multi-channel array controller simulations and using measured complex spherical polar data for a specific 3-way array cabinet are introduced. The specific array predictions are compared to direct sound frequency response measurements in a hall with arrays using the same cabinets as in the predictions. This comparison will indicate the degree of accuracy of array performance prediction that can be obtained by predictions based on measured complex spherical polar data for the array cabinets.

Finally, measurements and directive point source model 1 and 2 predictions are compared. The purpose of these comparisons is to find out if the direct point source is accurate enough to allow a general analysis of line array mid and high frequency responses and their dependency on the above mentioned factors.

## 2. FREQUENCY RESPONSES PREDICTIONS BY DIRECTIVE POINT SOURCE MODEL 1

The frequency response produced by the array at an observation point in the audience area is calculated by the following complex summation of the sound pressure contributions from the directive point sources that are modeling the array cabinets.

$$p_{res}^2(f) = \left[ k 10^{sens/20} \sum_{i=1}^n \frac{U_i}{r_i} R(\varphi_i, \theta_i) a(f) \cos(\Phi_i(f)) \right]^2 + \left[ k 10^{sens/20} \sum_{i=1}^n \frac{U_i}{r_i} R(\varphi_i, \theta_i) a(f) \sin(\Phi_i(f)) \right]^2 \quad (1)$$

$p_{res}^2$  is the mean square of the resulting sound pressure at a given position in the audience area,  $k$  is a constant,  $sens$  is the sensitivity in dB SPL/1m/2.83V of the  $n$  point source loudspeakers,  $U_i$  is the input voltage to point source  $i$ ,  $r_i$  is the distance from point source  $i$  to the observation point.

$R(\varphi_i, \theta_i)$ , which is the directivity function for point source  $i$  with  $\varphi_i$  and  $\theta_i$  as the spherical coordinates determining the direction to the observation point, has been modeled as

$$R(\varphi_i, \theta_i) = \frac{1 + \cos(\varphi_i) \cos(\theta_i)}{2 + m_\varphi \sin(\varphi_i) + m_\theta \sin(\theta_i)} \quad (2)$$

where  $m_\varphi$  and  $m_\theta$  are factors that determine the vertical and horizontal 6 dB beamwidth of the directive point source. The directivity function (2) models, for a given set of  $m_\varphi$  and  $m_\theta$  values, the array cabinets as constant directivity devices.

$a(f)$  is the attenuation of sound in air at frequency  $f$  given by

$$a(f) = \exp(-\alpha(f)r_i) \quad (3)$$

where the attenuation factor  $\alpha(f)$  is calculated according to [4].

$\Phi_i(f)$  is the total phase angle of point source  $i$ ,

$$\Phi_i(f) = \theta_{pi}(f) - 2\pi fr_i/c - 2\pi ft_{di} \quad (4)$$

where  $\theta_{pi}(f)$  is the phase response value of point source  $i$ ,  $c$  is the propagation velocity of sound,  $2\pi fr_i/c$  is the phase corresponding to the propagation delay  $r_i/c$  and  $t_{di}$  is a possible electrical delay of point source loudspeaker  $i$ . As predictions only are made up to 16 kHz there is no need to introduce a LP magnitude frequency response at high frequencies at or above 16 kHz. However, as a LP phase response may influence the complex pressure summation significantly below 16 kHz the following 2<sup>nd</sup> order LP phase response has been used to model the phase response of the directive point sources.

$$\theta_{pi}(f) = -\arctan\left(\frac{2\xi_i f_c f}{f_c^2 - f^2}\right) \quad (5)$$

The damping constant  $2\xi_i$  has been used to create a phase response depending on direction by setting

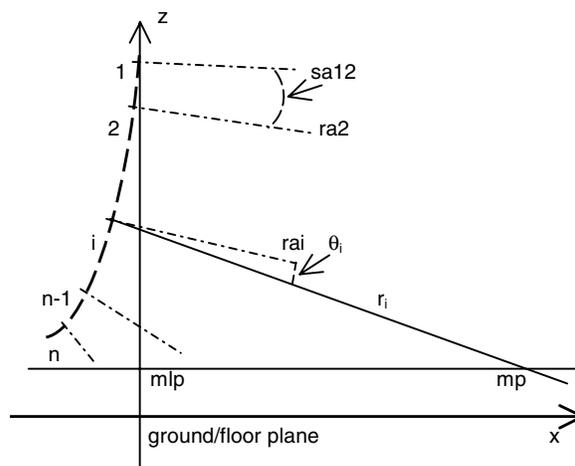
$$2\xi_i = 0.6 + 1.4 \frac{\varphi_i + \theta_i}{180} \quad (6)$$

Program code has been written based on equations (1) to (6), bandwidth smoothing procedure equations and trigonometric equations that calculate  $r_i$ ,  $\varphi_i$  and  $\theta_i$  based on the geometry of a given array as sketched in Figure 1.

The user input to the program is:

height of array cabinet, 2) position of reference point in array cabinet (= point of rotation for spherical data measurements = position of directive point source), 3) number of cabinets in array, 4) splay angles of array cabinets, 5) position ( $z$ -coordinate,  $x = 0$ ) of array top cabinet's upper front edge above ground/floor level ( $z = 0$ ), 6) position ( $z$ -coordinate) of microphone/listener plane below array bottom cabinet's lower front edge, 7) microphone position in the microphone plane ( $x$ -coordinate,  $y = 0$ ), 8) position ( $x$ -coordinate,  $x < 0$ ,  $y = 0$ ) of vertical axis for off-axis rotation of array, 9) off-axis angle, 10)  $sens$ , 11)  $U_i$ , 12)  $f_c$  of the phase response, 13)  $m_\varphi$  and  $m_\theta$ , 14)  $t_{di}$  and 15) percentage

relative humidity  $h_r$ . Static pressure and temperature have been fixed at 101.33 kPa and 20 °C.



**Figure 1: n point source array, sa12: splay angle between point source 1 and 2, ra2: reference axis of point source 2, mlp: microphone/listener head plane, mp: microphone position,  $\varphi_i$  is in a plane perpendicular to the  $zx$ -plane through  $rai$ .**

The program works with a basic frequency resolution of 1/36 octave as suggested in [5]. Predicted frequency responses are presented with 1/3 octave bandwidth smoothing.

The quantities in Table 1 define 2 rectilinear and 2 curved arrays. The curvature of the curved arrays is somewhere between the J array and the arithmetic spiral array described in [3]. The actual curvature appears from the sa-values in Table 1. The arrays are uniformly driven arrays, that is, arrays with constant  $U_i$  and  $t_{di} = 0$ .

Figure 2 to 5 show graphs of the predicted frequency responses with microphone position ( $x$ -coordinate,  $y = 0$ ) as parameter for the arrays defined in Table 1. The rectilinear arrays have been tilted to reduce the SPL variations over the range of microphone positions by selecting  $sa01 > 0$  ( $sa01$  is the angle between horizontal and the reference axis  $ra1$  of point source 1) and the rest of the splay angles equal to 0. However, it should be noted that the shown responses should be regarded as prediction examples and not as optimized design examples.

Experiments with the directive point source model 1 and comparisons with the directive point source model 2

Array	1	2	3	4
sa01 deg	8.5	4.2	5.25	4.25
sa12 -	0	0	1	1
sa23 -	0	0	1	1
sa34 -	0	0	1	1
sa45 -	0	0	1	1
sa56 -	0	0	2	1
sa67 -	0	0	2	2
sa78 -	0	0	2	2
sa89 -	-	0	-	3
sa910 -	-	0	-	4.5
sa1011 -	-	0	-	4.5
sa1112 -	-	0	-	4.5
Ver. beamwidth deg	7.5	7.5	7.5	7.5
Hor. beamwidth -	100	90	100	90
Cab. height m	0.24	0.49	0.24	0.49
Top z coord. m	4.6	9.8	4.5	9.8
Bottom z coord. m	2.6	3.9	2.6	4.1
mlp z coord. m	1.6	1.6	1.6	1.6
sens dB	103	108	103	108
$U_i$ V	2.83	2.83	2.83	2.83
$f_c$ kHz	12	12	12	12
$m_\phi$	1.7	1.7	1.7	1.7
$m_\theta$	35	35	35	35
$t_d$ msec	0	0	0	0
$h_r$ %	60	60	60	60

**Table 1. Quantities defining 2 rectilinear arrays and 2 curved arrays. The number of cabinets/point sources in an array appears from the number of sa-values.**

and in situ measurements on installed arrays (see paragraph 3) have shown that point source model 1 is somewhat sensitive to the use of the directivity function (selection of  $m_\phi$  and  $m_\theta$  in equation (2)) while it is less sensitive to changes in the phase response ( $f_c$  in equation (5)). Figures 3 and 5 are clearly illustrating the

importance of including air absorption in the model. The 4 sets of frequency responses are showing the benefit of using curved arrays. Also, they are showing the need for using for example HF shelving filters for the upper cabinets in a curved array to compensate for the air HF-absorption at long distances.

### 3. PREDICTED FREQUENCY RESPONSES BY DIRECTIVE POINT SOURCE MODEL 2 VERSUS MEASURED RESPONSES

The directive point source model 2 is like model 1 using complex summation of the pressure contributions from the individual cabinets in an array. However, instead of using models for the directivity and phase functions model 2 is using measured transfer functions for the LF, MF and HF sections of an array cabinet. The transfer functions have been measured in an anechoic chamber in the far field on a measuring sphere with an angular resolution of 5 degrees. Special software called DISPLAY - see [6] - has been developed to implement model 2 and process the measured complex spherical polar data. DISPLAY works with a frequency resolution of 1/36 octave and includes air absorption modeling and an exact model of a multi-channel array controller.

Frequency responses have been measured on an indoor installation of a 4 and an 8 cabinet array (array 3 in Table 1). The installation allowed 'free field' measurements up to a distance of 27m using MLSSA and B&K 2012 (TSR mode). Figures 6 to 9 are showing comparisons between the measured and the model 2 predicted frequency responses. The figures show that there is a satisfactory agreement between measurements and model 2 predictions. However, at longer distances and in the frequency range 8kHz to 16kHz there may appear differences between measurements and predictions that can amount to 6 to 8 dB. Taking the small wavelengths involved in this frequency range one should not expect a better agreement at these high frequencies. Line array cabinet to microphone alignment can probably not be adjusted with an accuracy better than a couple of centimeters. In addition time variances of the acoustic field at long and medium distances will be producing errors with MLS type measurements [7].

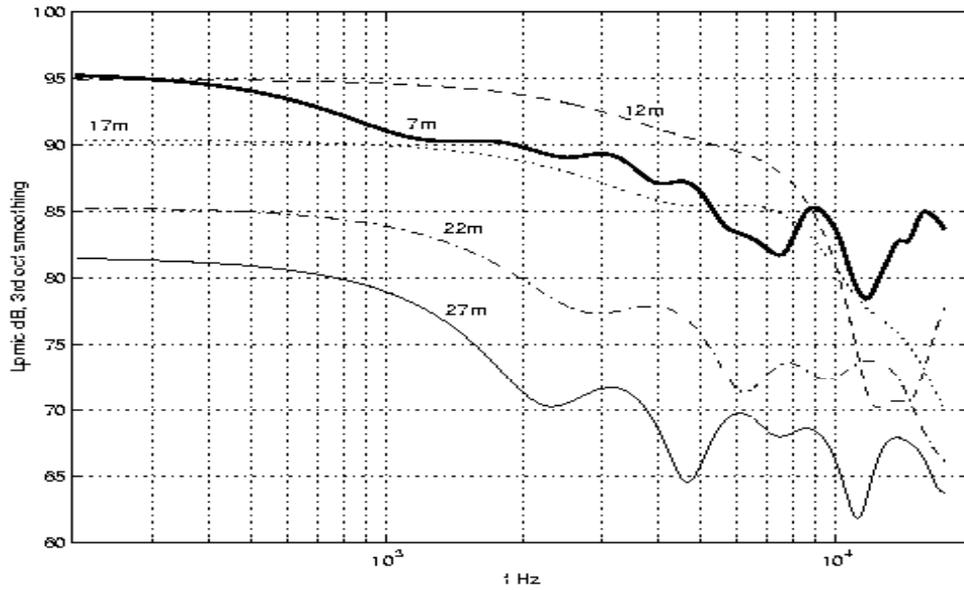


Figure 2: Array 1 (see Table 1) predicted frequency responses at different microphone positions.

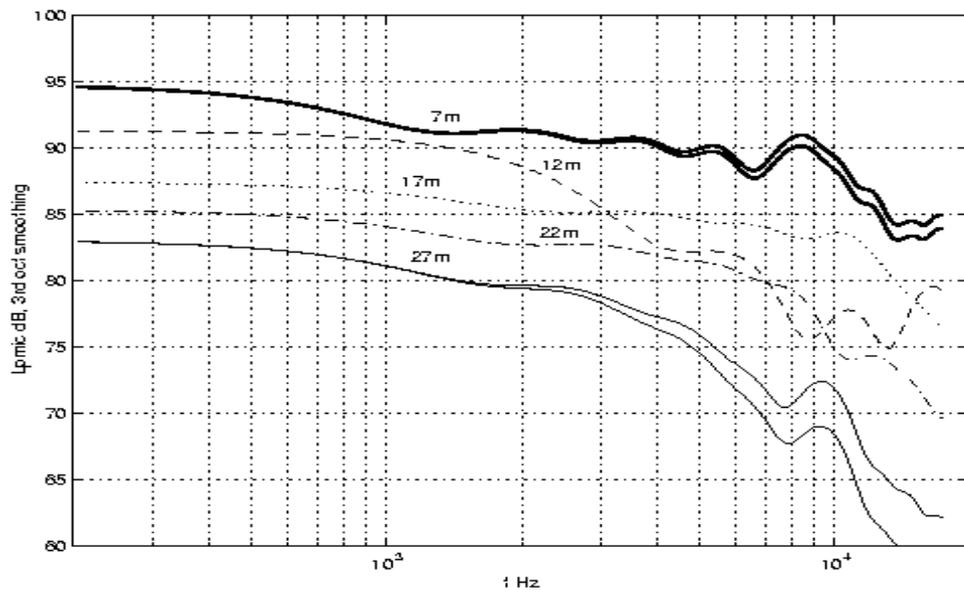


Figure 3: Array 2 (see Table 1) predicted frequency response at different microphone positions. The upper 7m and 27m curves are predicted with a relative humidity of 60% while the lower curves are predicted with 25%.

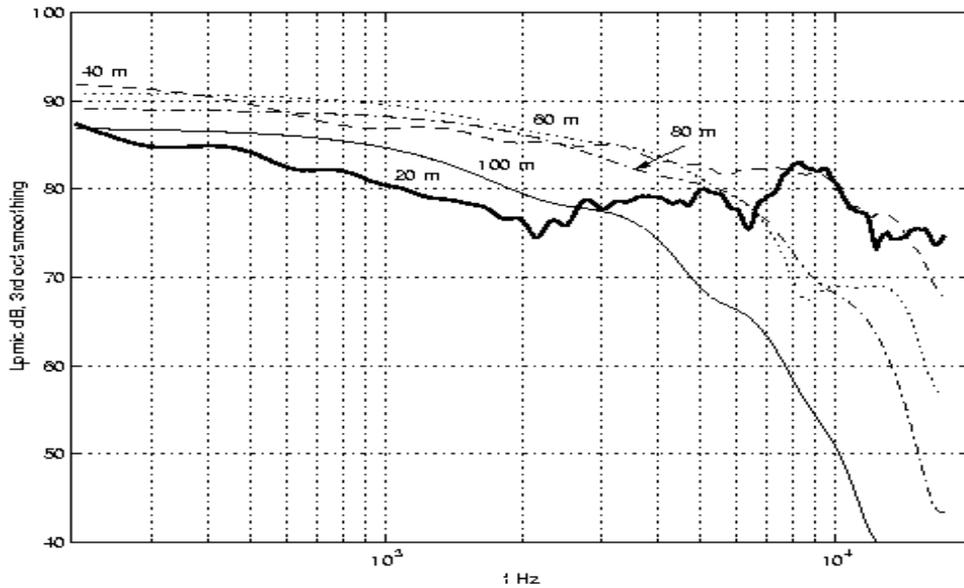


Figure 4: Array 3 (see Table 1) predicted frequency responses for different microphone positions.

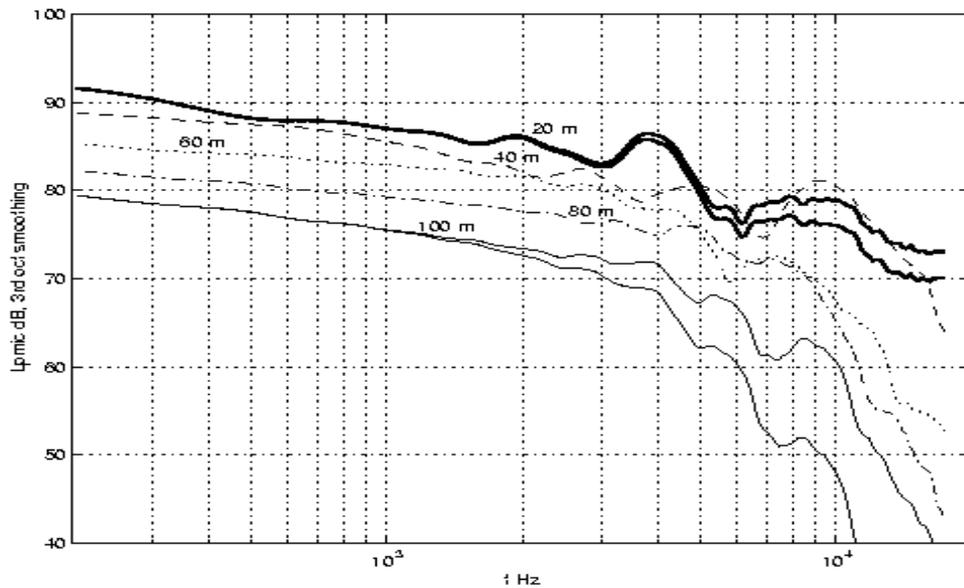


Figure 5: Array 4 (see Table 1) predicted frequency response at different microphone positions. The upper 20m and 100m curves are predicted with a relative humidity of 60% while the lower curves are predicted with 25%.

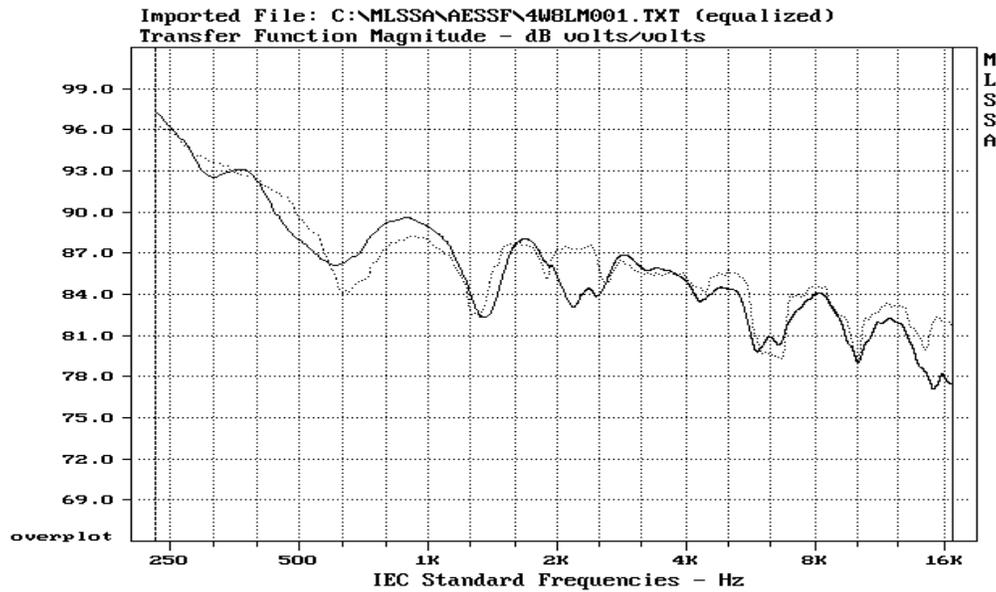


Figure 6: Measured (solid line) and model 2 predicted (dotted line) frequency responses at 2m on-axis from curved array with 4 small cabinets.

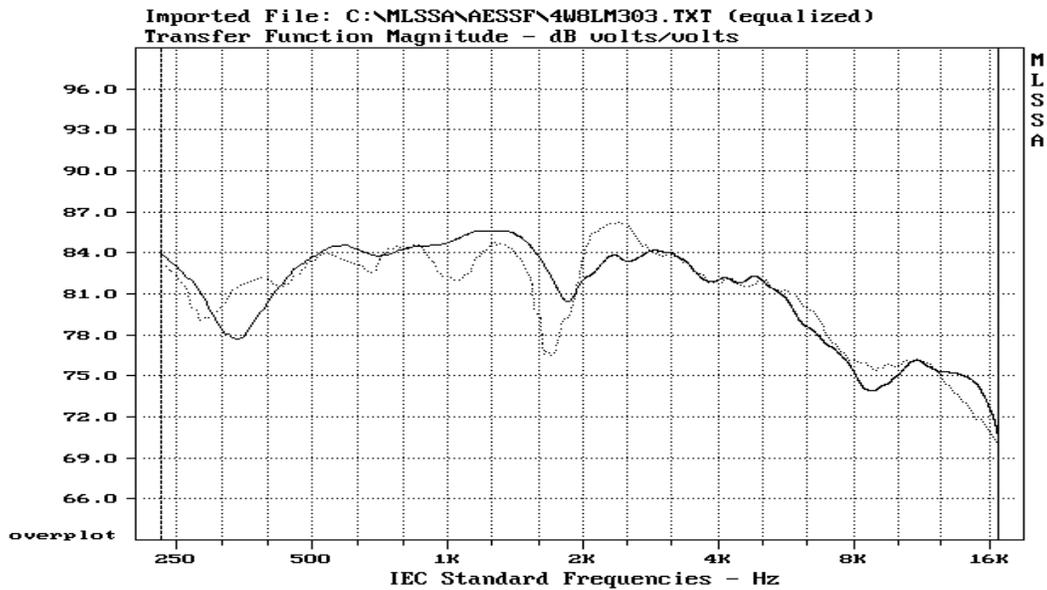


Figure 7: Measured (solid line) and model 2 predicted (dotted line) frequency responses at 12m, 30° off-axis from curved array with 4 small cabinets.

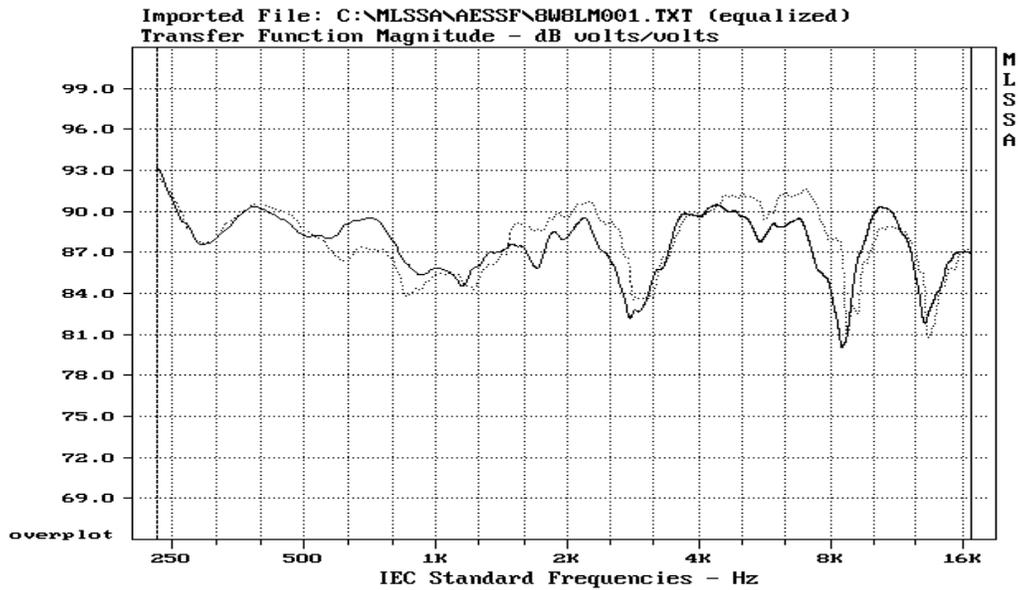


Figure 8: Measured (solid line) and model 2 predicted (dotted line) frequency responses at 2m, on-axis from curved array with 4 small cabinets.

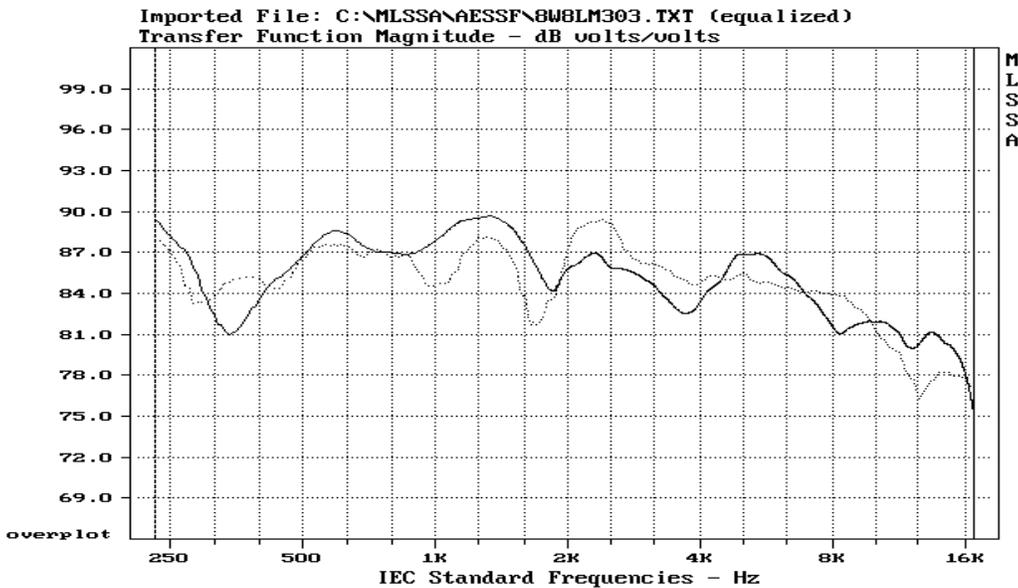


Figure 9: Measured (solid line) and model 2 predicted (dotted line) frequency responses at 12m, 30° off-axis from curved array with 4 small cabinets.

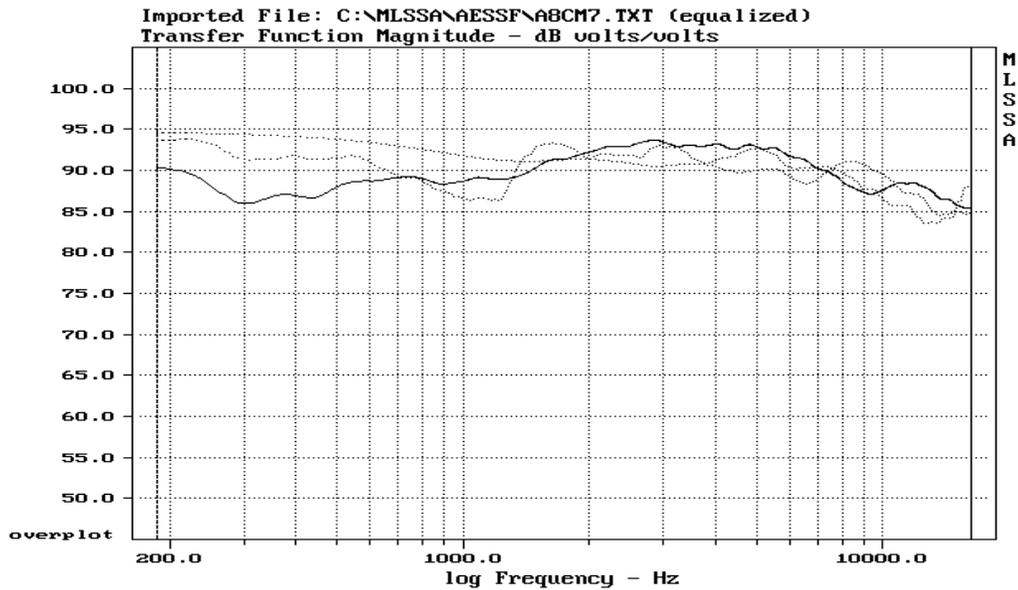


Figure 10: Measured (solid curve) and model 1 and model 2 predicted (dotted curves) frequency responses at 7m for array 2 - see Table 1 and Figure 3 to identify model 1 curves.

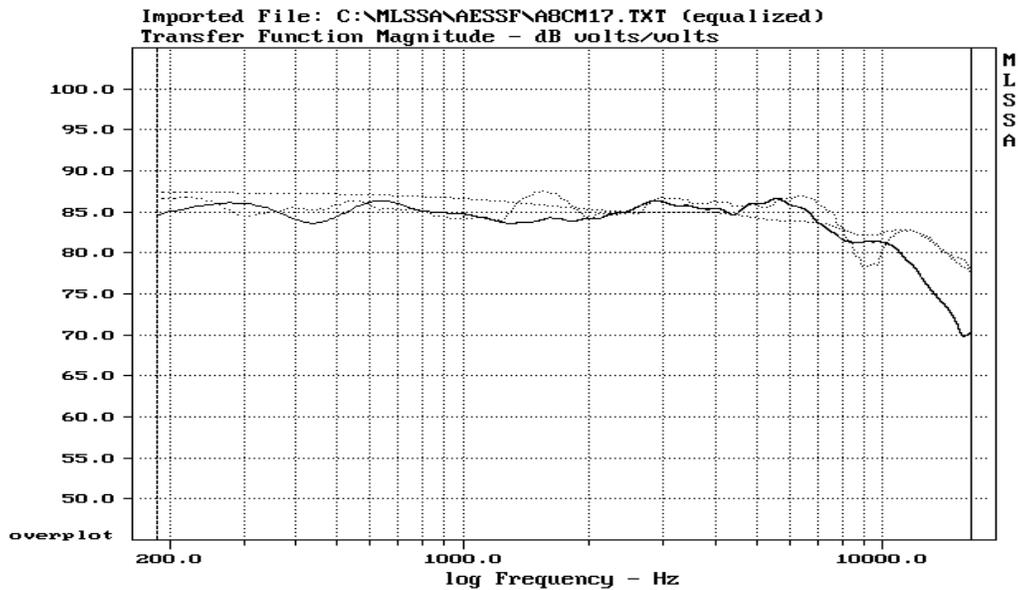


Figure 11: Measured (solid curve) and model 1 and model 2 predicted (dotted curves) frequency responses at 17m for array 2 - see Table 1 and Figure 3 to identify model 1 curves.

Figures 10 to 12 are comparing measurements, model 1 and model 2 predictions. The model 1 predictions can be identified and found in figure 3. Figures 11

and 12 illustrate the above mentioned HF discrepancies. Generally, there is a satisfactory agreement between the 3 sets of data.

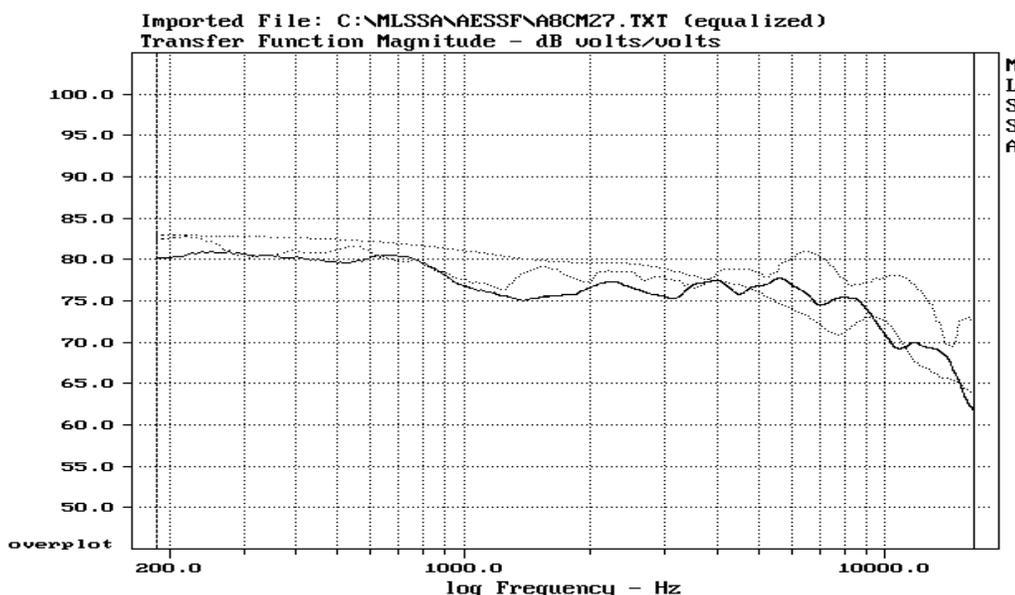


Figure 12: Measured (solid curve) and model 1 and model 2 predicted (dotted curves) frequency responses at 27m for array 2 - see Table 1 and Figure 3 to identify model 1 curves.

#### 4. CONCLUSION

The aim of these investigations was to see how accurate directive point source models can predict direct sound frequencies of arrays. The present investigations seem to point at an accuracy of  $\pm 3$  to 4 dB up to about 8 kHz and about  $\pm 6$  dB at higher frequencies for model 2 predictions. The accuracy of model 1 predictions is about 3dB less than the accuracy of model 2 predictions. Improvements may be achieved by using a frequency dependent directivity function that is closer to the directivity patterns of real array cabinets. An improved directivity model may allow a more general analysis of line array frequency responses. Future work to accomplish this will be directed at low and high frequencies. For low frequencies correction factors will be applied to the cabinet balloon measurement derived from full BEM of that cabinet in an array [8], whilst still maintaining the high speed of the directive point source model. For high frequencies a smaller

scale measurement, enabling more precise source /receiver positioning combined with a measurement system more tolerant of time variance, will be performed.

#### 5. ACKNOWLEDGEMENT

Thanks are due to Gabriel Radu for his work in implementing DISPLAY.

#### 6. REFERENCES

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